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AUTHOR Renick, Jobyann
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ABSTRACT

The optimal strategy at decision points in the singles game of squash played by North American rules is specific to the scoring system and the tie point at which the decision must be made. The actual decision should be based on the scoring potentials of the two players, from which projected outcome values of various length games are derived. One of the most interesting points from this analysis of tie points in squash is that the better player does not always increase his/her advantage by choosing the longer game. From the projected outcomes, the circumstances under which a player should choose the set and no-set options are determined, providing the optimal strategy for decision points in singles squash. This optimal strategy provides a player with the best possibility of winning a game. (Author/JS)

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Optimal Strategy at Decision Points in Singles Squash

Jobyann Renick
University of Washington

The process of making an intelligent decision involves serveral factors. First, one must consider what alternatives are available to choose among, and second, one must know whether or not there are any outside forces, human or otherwise, which may influence the outcomes of particular strategy choices. If there is another person with antithetical goals involved, a conflict situation is indicated in which outcomes are dependent on pairs of strategies. By matching pairs of strategies (one of his own against one of his opponent's) and determining the outcomes of these dyads, a player makes an intelligent decision by choosing a strategy which provides him with the best chance of success.

When one player in a two-person game makes a decision about the number of points which must be accumulated to win the game, it is generally thought that the better player should choose a longer game. The reason behind this strategical premise is that as the number of points is increased, variability is averaged out, and the probability of a "chance" victory is lessened. A poorer player or team might win a single point or game, but the probability of winning a greater number of points or a series of games is considerably less than the probability of winning one. For this reason, the World Series is a more accurate predictor of the superior team than the Super Bowl, and a tennis match is a better predictor than a single game or set.

A logical question arises as to whether or not this premise applies to other scoring games, particularly one like squash, where the structure of the game actually stipulates decision points in which one player must choose between a longer or a shorter game. The purpose of this paper is to frame this issue in a realistic manner for the singles game of squash and then determine the optimal

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strategy at decision points for the player who must choose between two options.

In the singles game of squash played by North American rules, a player must score a minimum of 15 points to win. Points are accumulated by winning any rally with the additional stipulation that a serving player who loses a rally also loses the right of service. Further, the rules stipulate that when the score is tied at (13-13) or (14-14), the first player who reached that score must choose between a "set" and "no-set" option. In both instances, the "no-set" option is a choice to end the game at 15 points, while the "set" option is a decision for a longer game. At the (13-13) ties, the "set" decision requires a player to acquire five additional points, while at (14-14) choosing the "set" decision means that the first player to win three points wins the game. From the above statements, it is clear the choice at (13-13) is between a two-point and a five-point game, and the decision at (14-14) is between a one-point and a three-point game.

Assuming consistency of play between the two players, the "better player" strategy described earlier would apply if the game progressed in the same manner as the score. The analysis would involve the following reasoning. Given two players, A and B, where B has "i" points and A has "j" points, written (i, j), the next score must be either (i+1,j) or (i, j+1). Other authors, ApSimon (1951, 1957) and Schutz and Kinsey (1975), have assumed that from any score point, (i,j), there is one probability, say "x", that the score will advance to (i+1,j). This implies the probability the score will advance to (i,j+1) is (1-x). Unfortunately, even though the score progresses in this manner, there are two ways it can progress in favor of one player, since a player can score either when serving or receiving. The probability a player will score when serving is not necessarily the same as the probability he will score when receiving. For example, against a given opponent, a player's serving game may be much stronger than his receiving game. While this difference may average out over a long period of play, the difference is an important factor in making a decision between the small number of points involved when the game is tied. Keeping this in mind, the problem may be formulated approximately in the following manner.

Suppose the two players are A and B where A is the player faced with the decision of what to do at one of the tie points. If A is serving, he will either score and retain service or he will lose a point and forfeit the right of service. Let P(A), the probability A will score when serving, be "a". It follows that the probability B will score when A serves is (1-a). Similarly, let P(B), the probability of B scoring while serving, be "b", implying the probability of A scoring while B is serving is (1-b). If A is to make the decision, B will serve first since he has just scored to tie the game and, therefore, retains the right of service.

Using this analysis, models were constructed of the different games a player might have to choose between at the two points (a one-, two-, three- and five-point game). From these, formulas in terms of the general case were derived for the win probability of the deciding player. For a one-point game, if A is the deciding player, the probability he will win is the probability B will not score when serving, (1-b). In a two-point game there are three ways A can win, and in terms of what the serving player does, these ways may be described as follows:

1. B loses a point, A wins a point.
2. B loses a point, A loses a point, B loses a point.
3. B wins a point, B loses a point, A wins a point.

The accompanying probabilities of these three ways A can win are the products of the singular events occurring, or $(1-b)a$, $(1-b)\ (1-a)\ (1-b)$ or $(1-b)^2\ (1-a)$ and $b(1-b)a$, respectively. The probability A will win a two-point game is the sum of these, or $(1-b)\ (a+(1-b)\ (1-a+ba)$. The formulas for the three-point and five-point games were derived in a similar fashion.[1]

By assigning different numerical values to $P(A)$ and $P(B)$, the formulas may be used to project the outcome of games requiring a certain number of points to win the game. These outcome values for the probability A wins an n-point game, $P(A_w)_n$, were calculated for different pairs of $P(A)$ and $P(B)$ from .10 to .90 with .10 intervals. It was felt these intervals were sufficient to differentiate while still being reasonable to handle. A computer was programmed to calculate these values.

Insert Table 1 about here

Table 1 shows the computed values for $P(A_w)1$ and $P(A_w)3$ and provide a means of determining the optimal strategy at the (14-14) tie where A is the deciding player. For any pair of values, if the outcomes are above the line, the player's better choice is not to set the game. Conversely, if the outcomes are below the line, the player should choose to set. More explicitly, the results may be stated as follows:

1. If $P(B) < .5$, the set option is better only if $P(A) \geq .5$.
2. If $P(B) > .5$, the set option is better only if $P(A) \geq .6$.
3. If $P(B) = .5 = P(A)$, there is no difference between the two options, and both players have an equal chance of winning.
4. If $P(A) \geq .6$, the set option is better regardless of the value of $P(B)$.
5. If $P(A) \leq .4$, the no-set option is better regardless of the value of $P(B)$.
6. If $P(A) = P(B)$, the no-set option is better if $P(A) \leq .5$, while the set option is better if $P(A) > .5$.

Insert Table 2 about here

Table 2 shows the computed values for $P(A_w)_3$ and $P(A_w)_5$,[2] and the line on this table also separates the no-set choice from the set choice. Since A is the deciding player, conclusions regarding optimal strategy at the (13-13) tie may be summarized as follows.

1. If $P(A) < .4$, the no-set option is always better.
2. If $P(A) = .4$, the set option is better only if $P(B) < .3$.
3. $P(A) = .5$, the set option is better when $P(B) < .5$, and the no-set is better when $P(B) \geq .5$.
4. If $P(A) > .5$ the set option is better when P(A) is at least greater than or equal to P(B).
5. If $P(A) \geq .8$, the set option is always better regardless of P(B).
6. If $P(A) = P(B)$, the no-set is better if $P(A) \leq .5$, while the set option is better if P(A)   .5.

General Conclusions and Discussion

The optimal strategy at decision points in the singles game of squash played by North American Rules is specific to the scoring system and the tie point at which the decision must be made. The actual decision should be based on the scoring potentials of the two players and the projected outcome values given in the paper.

One of the most interesting points from this analysis of tie points in squash is that the "better player" (when $P(A) > P(B)$ does not always increase his advantage by choosing the longer game. At the (14-14) tie, the deciding player should choose the shorter game if $P(A) < .5$, regardless of the P(B) value, and if $P(A) > .5$, he should choose to set even if his opponent is better. At the (13-13) tie, the notion that the better player should choose the longer game is generally true when $P(A) > .5$, but there are still exceptions. This is contrary to intuitive estimates of outcomes and indicates the importance of being able to accurately estimate the scoring potentials of both players.

While it would be a simple task for a person watching a game to keep track of scoring and actually calculate P(A) and P(B), this may not be feasible in more casual contests where no one is watching. For this reason, some general clues for estimating the serving potentials while playing might be helpful. The number of serves per service provides an excellent clue for this task. If a player has exactly one serve per service, his probability would be zero because this would mean he lost every serve. If he serves twice each service, his probability would be .5 since he would score on exactly half of them. Three serves per service would give a .67 probability of scoring when serving, and four serves per service would yield a .75 probability. For this reason, a player who has scored in streaks of four or more points at any time during the game can usually conclude that his scoring probability is at least .6, unless he has good evidence to the contrary. On the other hand, if a player has scored most of his points while receiving, his serving probability is likely to be less than .5. However, this player would not find himself in the position of having to make a decision at tie points unless playing against an equally poor opponent because the game would not reach a tie point.

The last observation leads to the consideration of the special case when players are of equal ability (i.e., when P(A) = P(B)). These pairs of players are much more likely to reach decision points than players with a great disparity in ability. While players of equal ability have roughly an equal chance of winning the entire game, they do not necessarily have an equal chance of winning the shorter game involved at decision points.

Insert Table 3 about here

Table 3 indicates the probability of the deciding player winning an n-point game against an equally competent opponent and can be used to derive appropriate strategies at decision points. When both players have an equally poor chance of scoring when serving (when the probability is less than .5), the deciding player is wise to choose the shorter game. Essentially, this allows his equally poor opponent to lose the game before he does. On the other hand, when both players have a better than 50-50 chance of scoring when serving, the deciding player should choose the longer game. This gives him a better chance of getting an opportunity to serve and use his ability to win the game.

P(A)		P(B) .10	.20	.30	.40	.50	.60	.70	.80	.90
.10	$P(A_w)_3$	.78	.59	.43	.30	.20	.12	.07	.03	.01
	$P(A_w)_1$	.90	.80	.70	.60	.50	.40	.30	.20	.10
.20	$P(A_w)_3$	.82	.65	.51	.38	.27	.18	.11	.06	.02
	$P(A_w)_1$	.90	.80	.70	.60	.50	.40	.30	.20	.10
.30	$P(A_w)_3$	.86	.72	.58	.46	.35	.25	.16	.09	.04
	$P(A_w)_1$	.90	.80	.70	.60	.50	.40	.30	.20	.10
.40	$P(A_w)_3$	.89	.77	.66	.54	.42	.32	.22	.13	.06
	$P(A_w)_1$	.90	.80	.70	.60	.50	.40	.30	.20	.10
.50	$P(A_w)_3$	.92	.82	.72	.61	.50	.39	.28	.18	.08
	$P(A_w)_1$	.90	.80	.70	.60	.50	.40	.30	.20	.10
.60	$P(A_w)_3$	.94	.87	.78	.68	.57	.46	.34	.23	.11
	$P(A_w)_1$	.90	.80	.70	.60	.50	.40	.30	.20	.10
.70	$P(A_w)_3$	.96	.91	.84	.75	.65	.54	.42	.28	.14
	$P(A_w)_1$	.90	.80	.70	.60	.50	.40	.30	.20	.10
.80	$P(A_w)_3$	.98	.94	.89	.82	.72	.62	.49	.35	.18
	$P(A_w)_1$	.90	.80	.70	.60	.50	.40	.30	.20	.10
.90	$P(A_w)_3$	.99	.97	.93	.88	.80	.70	.57	.41	.22
	$P(A_w)_1$	.90	.80	.70	.60	.50	.40	.30	.20	.10

TABLE I $P(A_w)$ from Squash (14-14) tie

P(A)		P(B) .10	.20	.30	.40	.50	.60	.70	.80	.90
.10	$P(A_w)_5$	.70	.47	.30	.18	.10	.05	.02	.01	.00
	$P(A_w)_2$	.83	.67	.53	.41	.30	.21	.13	.07	.03
.20	$P(A_w)_5$	.78	.59	.42	.29	.18	.10	.05	.02	.00
	$P(A_w)_2$	.85	.70	.57	.46	.35	.26	.17	.10	.05
.30	$P(A_w)_5$	.85	.69	.54	.40	.28	.17	.10	.04	.01
	$P(A_w)_2$	.86	.74	.62	.50	.40	.30	.22	.14	.06
.40	$P(A_w)_5$	.90	.78	.65	.51	.38	.26	.16	.08	.03
	$P(A_w)_2$	.88	.77	.66	.55	.45	.35	.26	.17	.08
.50	$P(A_w)_5$	.94	.85	.74	.62	.49	.36	.24	.	.05
	$P(A_w)_5$	.90	.80	.70	.60	.50	.40	.30	.20	.10
.60	$P(A_w)_5$	.96	.91	.83	.72	.60	.47	.33	.20	.08
	$P(A_w)_2$	.92	.83	.74	.65	.55	.45	.34	.23	.12
.70	$P(A_w)_5$	.98	.95	.89	.81	.71	.58	.44	.28	.13
	$P(A_w)_2$	.94	.86	.78	.70	.60	.50	.38	.26	.14
.80	$P(A_w)_5$	.99	.98	.94	.89	.81	.70	.56	.39	.20
	$P(A_w)_2$	.95	.90	.83	.74	.65	.54	.43	.30	.15
.90	$P(A_w)_5$	1.00	.99	.98	.95	.90	.81	.69	.52	.29
	$P(A_w)_2$	.97	.93	.87	.79	.70	.59	.47	.30	.17

TABLE 2 $P(A_w)$ from Squash (13-13) tie

Notes

1. The formulas for $P(A_w)_3$ and $P(A_w)_5$ are available on request.
2. In some instances, a player has three options at the (13-13) tie:

   a. Not set- a two-point game

   b. Set to three- a three-point game

   c. Set to five- a five-point game

However, the option of setting to three never produces a better outcome for the deciding player than one of the others and, hence, should never be used. For this reason, this option is not included in this paper and could be excluded from the rules of singles squash since it serves no real purpose.

References

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Schutz, R. and Kinsey, W. (1975). A Comparison of North American and International Squash Scoring Systems - A Computer Simulation. Unpublished manuscript. University of British Columbia.

$P(A)$	.1	.2	.3	.4	.5	.9	.7	.8	.9
$P(A_w)_1$	.90	.80	.70	.60	.50	.40	.30	.20	.10
$P(A_w)_2$	.83	.70	.62	.55	.50	[illegible]	[illegible]	.30	.17
$P(A_w)_3$	.78	.63	.58	.54	.50	.46	.42	.35	.22
$P(A_w)_5$	.70	.59	.54	.51	.49	.47	.44	.35	.29

TABLE 3. $P(A_w)n$ in Squash when $P(A) = P(B)$

Badminton

	.10	.20	.30	.40	.50	.60	.70	.80	.90
$P(A_w)_5$	.36	.36	.36	.36	.351	.35	.33	.30	.23
$P(A_w)_3$	.49	.48	.47	.45	.43	.41	.37	.31	.21
$P(A_w)_2$	.49	.47	.45	.43	.41	.37	.33	.26	.16
$P(A_w)_1$	.47	.44	.41	.38	.33	.29	.23	.17	.09

TABLE: $P(A_w)_n$ when $P(A) = P(B)$

Badminton

P(A)		P(B) .10	.20	.30	.40	.50	.60	.70	.80	.90
.10	5	.36	.08	.02	.01	.00	.00	.00	.00	.00
	2	.49	.23	.12	.07	.04	.02	.01	.01	.00
.20	5	.77	.36	.15	.06	.02	.01	.00	.00	.00
	2	.74	.47	.30	.19	.12	.07	.04	.02	.01
.30	5	.91	.63	.36	.19	.09	.04	.02	.01	.00
	2	.85	.64	.45	.32	.22	.14	.09	.05	.02
.40	5	.97	.79	.56	.36	.21	.11	.06	.02	.01
	2	.91	.74	.58	.43	.31	.22	.14	.08	.04
.50	5	.99	.89	.72	.53	.35	.22	.12	.06	.02
	2	.94	.81	.67	.53	.41	.30	.20	.12	.05
.60	5	.99	.94	.83	.67	.50	.35	.21	.11	.04
	2	.97	.86	.75	.62	.49	.37	.26	.16	.08
.70	5	1.00	.97	.90	.79	.64	.49	.33	.19	.08
	2	.97	.90	.80	.69	.57	.45	.33	.21	.10
.80	5	1.00	.99	.95	.88	.77	.63	.47	.30	.14
	2	.98	.93	.85	.75	.64	.52	.39	.26	.13
.90	5	1.00	.99	.98	.94	.86	.76	.61	.44	.23
	2	.99	.95	.88	.80	.70	.58	.45	.31	.16

Table: $P(A_w)$ from (13-13) tie

Badminton (14-14) tie

P(A)		P(B) .10	.20	.30	.40	.50	.60	.70	.80	.90
.10	$P(A_w)_3$	.49	.18	.07	.03	.02	.01	.00	.00	.00
	$P(A_w)_1$	.47	.29	.19	.13	.09	.06	.07	.02	.01
.20	$P(A_w)_3$	.80	.48	.07	.15	.08	.04	.02	.01	.00
	$P(A_w)_1$	.64	.44	.32	.23	.17	.12	.08	.05	.02
.30	$P(A_w)_3$	.91	.68	.49	.30	.19	.11	.06	.03	.01
	$P(A_w)_1$	.73	.55	.41	.31	.23	.17	.11	.07	.03
.40	$P(A_w)_3$	.96	.81	.62	.43	.31	.20	.12	.06	.02
	$P(A_w)_1$	.78	.62	.48	.38	.29	.21	.15	.09	.04
.50	$P(A_w)_3$	.98	.88	.74	.53	.43	.30	.19	.11	.05
	$P(A_w)_1$	.82	.67	.54	.43	.33	.25	.18	.11	.05
.60	$P(A_w)_3$	.99	.93	.82	.69	.55	.41	.28	.17	.07
	$P(A_w)_1$	.84	.71	.58	.47	.38	.29	.20	.13	.06
.70	$P(A_w)_3$	.99	.96	.88	.78	.65	.51	.37	.24	.11
	$P(A_w)_1$	.86	.74	.62	.51	.41	.32	.23	.15	.07
.80	$P(A_w)_3$	1.00	.97	.92	.84	.74	.61	.47	.31	.16
	$P(A_w)_1$	.88	.76	.65	.55	.44	.33	.26	.17	.08
.90	$P(A_w)_3$	1.00	.98	.98	.90	.81	.70	.56	.40	.21
	$P(A_w)_1$	.89	.78	.68	.57	.47	.38	.28	.18	.09

Table: $P(A_w)$ from (14-14) tie